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[Theorems and Characteristic Matrix Functions](#) [Evolution of Biological Systems in Random Media: Limit Theorems and Stability](#)

This book takes a comprehensive look at mean value theorems and their connection with functional equations. Besides the traditional Lagrange and Cauchy mean value theorems, it covers the Pompeiu and the Flett mean value theorems as well as extension to higher dimensions and the complex plane. Furthermore the reader is introduced to the field of functional equations through equations that arise in connection with the many mean value theorems discussed. Arrow Impossibility Theorems is a 10-chapter text that describes existing impossibility theorems. This book explores a number of formalizations of ethical constraints of the theorems. After an introduction to the framework and notation for Arrow impossibility theorems, this book goes on discussing some concepts and an apparatus of relations among those concepts which are important for the theorems. Other chapters present some impossibility results that serve to point out serious difficulties in some plausible escape routes from the theorems of earlier chapters. The final chapter describes important areas of research that have arisen in the collective choice field in the transition away from studying the conditions of Arrow's theorem alone to the totality of all impossibility theorems. This book is intended primarily for economists. Structural Theorems and Their Applications is an account of the various structural theorems and their applications. Topics covered range from the principles of superposition to virtual work and energy concepts, calculation of deflections, and analysis of indeterminate structures using the compatibility and equilibrium methods. Reciprocal theorems and theorems of plastic analysis for plane frames are also discussed. This book is comprised of eight chapters and begins with an overview of the problems of structural analysis and the

importance of the principle of virtual work in this regard, followed by an analysis of the principles of superposition. The next chapter is devoted to virtual work and energy concepts such as strain energy and complementary energy. The principle of virtual work is used in the subsequent chapters as the basis for all of the indirect methods of structural analysis described in the text, including the analysis of indeterminate structures using the compatibility method and the equilibrium method. The principle of virtual work is also used to prove the reciprocal theorems and to establish the various theorems of plastic and incremental collapse for framed structures. This monograph will be of interest to mechanical and structural engineers. The authors present a concise but complete exposition of the mathematical theory of stable convergence and give various applications in different areas of probability theory and mathematical statistics to illustrate the usefulness of this concept. Stable convergence holds in many limit theorems of probability theory and statistics - such as the classical central limit theorem - which are usually formulated in terms of convergence in distribution. Originated by Alfred Rényi, the notion of stable convergence is stronger than the classical weak convergence of probability measures. A variety of methods is described which can be used to establish this stronger stable convergence in many limit theorems which were originally formulated only in terms of weak convergence. Naturally, these stronger limit theorems have new and stronger consequences which should not be missed by neglecting the notion of stable convergence. The presentation will be accessible to researchers and advanced students at the master's level with a solid knowledge of measure theoretic probability. The author develops the limit relations between the errors of polynomial approximation in weighted metrics and apply them to various problems in approximation theory such as asymptotically best constants, convergence of polynomials, approximation of individual functions, and multidimensional limit theorems of polynomial approximation. In 1915 and 1916 Emmy Noether was asked by Felix Klein and David Hilbert to assist them in understanding issues involved in any attempt to formulate a general

theory of relativity, in particular the new ideas of Einstein. She was consulted particularly over the difficult issue of the form a law of conservation of energy could take in the new theory, and she succeeded brilliantly, finding two deep theorems. But between 1916 and 1950, the theorem was poorly understood and Noether's name disappeared almost entirely. People like Klein and Einstein did little more than mention her name in the various popular or historical accounts they wrote. Worse, earlier attempts which had been eclipsed by Noether's achievements were remembered, and sometimes figure in quick historical accounts of the time. This book carries a translation of Noether's original paper into English, and then describes the strange history of its reception and the responses to her work. Ultimately the theorems became decisive in a shift from basing fundamental physics on conservation laws to basing it on symmetries, or at the very least, in thoroughly explaining the connection between these two families of ideas. The real significance of this book is that it shows very clearly how long it took before mathematicians and physicists began to recognize the seminal importance of Noether's results. This book is thoroughly researched and provides careful documentation of the textbook literature. Kosmann-Schwarzbach has thus thrown considerable light on this slow dance in which the mathematical tools necessary to study symmetry properties and conservation laws were apparently provided long before the orchestra arrives and the party begins. This is a new book in biomathematics, which includes new models of stochastic non-linear biological systems and new results for these systems. These results are based on the new results for non-linear difference and differential equations in random media. This book contains: -New stochastic non-linear models of biological systems, such as biological systems in random media: epidemic, genetic selection, demography, branching, logistic growth and predator-prey models; -New results for scalar and vector difference equations in random media with applications to the stochastic biological systems in 1); -New results for stochastic non-linear biological systems, such as averaging, merging, diffusion approximation, normal deviations and stability; -New approach to the study of stochastic

biological systems in random media such as random evolution approach. This monograph studies the problem of characterizing canonical metrics on Hermitian locally symmetric manifolds X of non-compact/compact types in terms of curvature conditions. The proofs of these metric rigidity theorems are applied to the study of holomorphic mappings between manifolds X of the same type. Moreover, a dual version of the generalized Frankel Conjecture on characterizing compact Kähler manifolds are also formulated. Presenting the first unified treatment of limit theorems for multiple sums of independent random variables, this volume fills an important gap in the field. Several new results are introduced, even in the classical setting, as well as some new approaches that are simpler than those already established in the literature. In particular, new proofs of the strong law of large numbers and the Hajek-Renyi inequality are detailed. Applications of the described theory include Gibbs fields, spin glasses, polymer models, image analysis and random shapes. Limit theorems form the backbone of probability theory and statistical theory alike. The theory of multiple sums of random variables is a direct generalization of the classical study of limit theorems, whose importance and wide application in science is unquestionable. However, to date, the subject of multiple sums has only been treated in journals. The results described in this book will be of interest to advanced undergraduates, graduate students and researchers who work on limit theorems in probability theory, the statistical analysis of random fields, as well as in the field of random sets or stochastic geometry. The central topic is also important for statistical theory, developing statistical inferences for random fields, and also has applications to the sciences, including physics and chemistry. Kurt Godel, the greatest logician of our time, startled the world of mathematics in 1931 with his Theorem of Undecidability, which showed that some statements in mathematics are inherently "undecidable." His work on the completeness of logic, the incompleteness of number theory, and the consistency of the axiom of choice and the continuum theory brought him further worldwide fame. In this introductory volume, Raymond Smullyan, himself a well-known logician, guides the reader through the fascinating

world of Godel's incompleteness theorems. The level of presentation is suitable for anyone with a basic acquaintance with mathematical logic. As a clear, concise introduction to a difficult but essential subject, the book will appeal to mathematicians, philosophers, and computer scientists. Combinatorial reciprocity is a very interesting phenomenon, which can be described as follows: A polynomial, whose values at positive integers count combinatorial objects of some sort, may give the number of combinatorial objects of a different sort when evaluated at negative integers (and suitably normalized). Such combinatorial reciprocity theorems occur in connections with graphs, partially ordered sets, polyhedra, and more. Using the combinatorial reciprocity theorems as a leitmotif, this book unfolds central ideas and techniques in enumerative and geometric combinatorics. Written in a friendly writing style, this is an accessible graduate textbook with almost 300 exercises, numerous illustrations, and pointers to the research literature. Topics include concise introductions to partially ordered sets, polyhedral geometry, and rational generating functions, followed by highly original chapters on subdivisions, geometric realizations of partially ordered sets, and hyperplane arrangements. This text and software package introduces readers to automated theorem proving, while providing two approaches implemented as easy-to-use programs. These are semantic-tree theorem proving and resolution-refutation theorem proving. The early chapters introduce first-order predicate calculus, well-formed formulae, and their transformation to clauses. Then the author goes on to show how the two methods work and provides numerous examples for readers to try their hand at theorem-proving experiments. Each chapter comes with exercises designed to familiarise the readers with the ideas and with the software, and answers to many of the problems. A clear and accessible treatment of Gödel's famous, intriguing, but much misunderstood incompleteness theorems, extensively revised in a second edition. This book provides a self-contained treatment of two of the main problems of multiparameter spectral theory: the existence of eigenvalues and the expansion in series of eigenfunctions. The results are first obtained in abstract Hilbert spaces and then applied to integral

operators and differential operators. Special attention is paid to various definiteness conditions which can be imposed on multiparameter eigenvalue problems. The reader is not assumed to be familiar with multiparameter spectral theory but should have some knowledge of functional analysis, in particular of Brower's degree of maps. The noncommutative versions of fundamental classical results on the almost sure convergence in L2-spaces are discussed: individual ergodic theorems, strong laws of large numbers, theorems on convergence of orthogonal series, of martingales of powers of contractions etc. The proofs introduce new techniques in von Neumann algebras. The reader is assumed to master the fundamentals of functional analysis and probability. The book is written mainly for mathematicians and physicists familiar with probability theory and interested in applications of operator algebras to quantum statistical mechanics. This book provides a primary resource in basic fixed-point theorems due to Banach, Brouwer, Schauder and Tarski and their applications. Key topics covered include Sharkovsky's theorem on periodic points, Thron's results on the convergence of certain real iterates, Shield's common fixed theorem for a commuting family of analytic functions and Bergweiler's existence theorem on fixed points of the composition of certain meromorphic functions with transcendental entire functions. Generalizations of Tarski's theorem by Merrifield and Stein and Abian's proof of the equivalence of Bourbaki-Zermelo fixed-point theorem and the Axiom of Choice are described in the setting of posets. A detailed treatment of Ward's theory of partially ordered topological spaces culminates in Sherrer fixed-point theorem. It elaborates Manka's proof of the fixed-point property of arcwise connected hereditarily unicoherent continua, based on the connection he observed between set theory and fixed-point theory via a certain partial order. Contraction principle is provided with two proofs: one due to Palais and the other due to Barranga. Applications of the contraction principle include the proofs of algebraic Weierstrass preparation theorem, a Cauchy-Kowalevsky theorem for partial differential equations and the central limit theorem. It also provides a proof of the converse of the contraction principle due to Jachymski, a

proof of fixed point theorem for continuous generalized contractions, a proof of Browder-Gohde-Kirk fixed point theorem, a proof of Stallings' generalization of Brouwer's theorem, examine Caristi's fixed point theorem, and highlights Kakutani's theorems on common fixed points and their applications. The classical result for uniqueness in elasticity theory is due to Kirchhoff. It states that the standard mixed boundary value problem for a homogeneous isotropic linear elastic material in equilibrium and occupying a bounded three-dimensional region of space possesses at most one solution in the classical sense, provided the Lamé and shear moduli, λ and μ respectively, obey the inequalities $(3\lambda + 2\mu) > 0$ and $\mu > 0$. In linear elastodynamics the analogous result, due to Neumann, is that the initial-mixed boundary value problem possesses at most one solution provided the elastic moduli satisfy the same set of inequalities as in Kirchhoff's theorem. Most standard textbooks on the linear theory of elasticity mention only these two classical criteria for uniqueness and neglect altogether the abundant literature which has appeared since the original publications of Kirchhoff. To remedy this deficiency it seems appropriate to attempt a coherent description of the various contributions made to the study of uniqueness in elasticity theory in the hope that such an exposition will provide a convenient access to the literature while at the same time indicating what progress has been made and what problems still await solution. Naturally, the continuing announcement of new results thwarts any attempt to provide a complete assessment. Apart from linear elasticity theory itself, there are several other areas where elastic uniqueness is significant. This volume is devoted to generalizations of the classical Birkhoff and von Neumann ergodic theorems to semigroup representations in Banach spaces, semigroup actions in measure spaces, homogeneous random fields and random measures on homogeneous spaces. The ergodicity, mixing and quasimixing of semigroup actions and homogeneous random fields are considered as well. In particular homogeneous spaces, on which all homogeneous random fields are quasimixing are introduced and studied (the n -dimensional Euclidean and Lobachevsky spaces with $n \geq 2$, and all simple Lie groups with finite centre are examples of such spaces. Also

dealt with are applications of general ergodic theorems for the construction of specific informational and thermodynamical characteristics of homogeneous random fields on amenable groups and for proving general versions of the McMillan, Breiman and Lee-Yang theorems. A variational principle which characterizes the Gibbsian homogeneous random fields in terms of the specific free energy is also proved. The book has eight chapters, a number of appendices and a substantial list of references. For researchers whose works involves probability theory, ergodic theory, harmonic analysis, measure theory and statistical Physics. Comparison Theorems in Riemannian Geometry The objective of the present edition of this monograph is the same as that of earlier editions, namely, to provide readers with some mathematical maturity a rigorous and modern introduction to the ideas and principal theorems of probabilistic information theory. It is not necessary that readers have any prior knowledge whatever of information theory. The rapid development of the subject has had the consequence that any one book can now cover only a fraction of the literature. The latter is often written by engineers for engineers, and the mathematical reader may have some difficulty with it. The mathematician who understands the content and methods of this monograph should be able to read the literature and start on research of his own in a subject of mathematical beauty and interest. The present edition differs from the second in the following: Chapter 6 has been completely replaced by one on arbitrarily varying channels. Chapter 7 has been greatly enlarged. Chapter 8 on semi-continuous channels has been drastically shortened, and Chapter 11 on sequential decoding completely removed. The new Chapters 11-15 consist entirely of material which has been developed only in the last few years. The topics discussed are rate distortion, source coding, multiple access channels, and degraded broadcast channels. Even the specialist will find a new approach in the treatment of these subjects. Many of the proofs are new, more perspicuous, and considerably shorter than the original ones. The book is devoted to limit theorems for nonconventional sums and arrays. Asymptotic behavior of such sums were first studied in ergodic theory but recently it turned out that main limit theorems of

probability theory, such as central, local and Poisson limit theorems can also be obtained for such expressions. In order to obtain sufficiently general local limit theorem, we develop also thermodynamic formalism type results for random complex operators, which is one of the novelties of the book. Contents: Nonconventional Limit Theorems: Stein's Method for Nonconventional Sums Local Limit Theorem Nonconventional Arrays Random Transformations Thermodynamic Formalism for Random Complex Operators: Ruelle-Perron-Frobenius Theorem via Cone Contractions Application to Random Locally Expanding Covering Maps Pressure, Asymptotic Variance and Complex Gibbs Measures Application to Random Complex Integral Operators Fiberwise Limit Theorems Readership: Advanced graduate students and researchers in probability theory and stochastic processes and dynamical systems and ergodic theory. Keywords: Limit Theorems;Nonconventional Sums;Nonconventional Arrays;Stochastic Processes;Dynamical Systems;Stein's Method;Martingale Approximation;Thermodynamic Formalism;Strong Law of Large Numbers;Central;Local and Poisson Limit TheoremsReview: Key Features: The results in the book are new and never appeared before, Prof. Yuri Kifer is a well-known researcher in probability and dynamical systems, he published several books and more than 130 papers and he initiated the research on nonconventional limit theorems in the last decade The subject of this book is probabilistic number theory. In a wide sense probabilistic number theory is part of the analytic number theory, where the methods and ideas of probability theory are used to study the distribution of values of arithmetic objects. This is usually complicated, as it is difficult to say anything about their concrete values. This is why the following problem is usually investigated: given some set, how often do values of an arithmetic object get into this set? It turns out that this frequency follows strict mathematical laws. Here we discover an analogy with quantum mechanics where it is impossible to describe the chaotic behaviour of one particle, but that large numbers of particles obey statistical laws. The objects of investigation of this book are Dirichlet series, and, as the title shows, the main attention is devoted to the Riemann zeta-function.

In studying the distribution of values of Dirichlet series the weak convergence of probability measures on different spaces (one of the principle asymptotic probability theory methods) is used. The application of this method was launched by H. Bohr in the third decade of this century and it was implemented in his works together with B. Jessen. Further development of this idea was made in the papers of B. Jessen and A. Wintner, V. Borchsenius and B. The present book is the first ever published in which a new type of eigenvalue problem is studied, one that is very useful for applications: eigenvalue problems related to hemivariational inequalities, i.e. involving nonsmooth, nonconvex, energy functions. New existence, multiplicity and perturbation results are proved using three different approaches: minimization, minimax methods and (sub)critical point theory. Nonresonant and resonant cases are studied both for static and dynamic problems and several new qualitative properties of the hemivariational inequalities are obtained. Both simple and double eigenvalue problems are studied, as well as those constrained on the sphere and those which are unconstrained. The book is self-contained, is written with the utmost possible clarity and contains highly original results. Applications concerning new stability results for beams, plates and shells with adhesive supports, etc. illustrate the theory. Audience: applied and pure mathematicians, civil, aeronautical and mechanical engineers. This is the only book that deals comprehensively with fixed point theorems overall of mathematics. Their importance is due, as the book demonstrates, to their wide applicability. Beyond the first chapter, each of the other seven can be read independently of the others so the reader has much flexibility to follow his/her own interests. The book is written for graduate students and professional mathematicians and could be of interest to physicists, economists and engineers. This monograph originated with a course of lectures on information theory which I gave at Cornell University during the academic year 1958-1959. It has no pretensions to exhaustiveness, and, indeed, no pretensions at all. Its purpose is to provide, for mathematicians of some maturity, an easy introduction to the ideas and principal known theorems of a certain body of coding theory. This

purpose will be amply achieved if the reader is enabled, through his reading, to read the (sometimes obscurely written) literature and to obtain results of his own. The theory is obviously in a rapid stage of development; even while this monograph was in manuscript several of its readers obtained important new results. The first chapter is introductory and the subject matter of the monograph is described at the end of the chapter. There does not seem to be a uniquely determined logical order in which the material should be arranged. In determining the final arrangement I tried to obtain an order which makes reading easy and yet is not illogical. I can only hope that the resultant compromises do not earn me the criticism that I failed on both counts. There are a very few instances in the monograph where a stated theorem is proved by a method which is based on a result proved only later. This monograph presents necessary and sufficient conditions for completeness of the linear span of eigenvectors and generalized eigenvectors of operators that admit a characteristic matrix function in a Banach space setting. Classical conditions for completeness based on the theory of entire functions are further developed for this specific class of operators. The classes of bounded operators that are investigated include trace class and Hilbert-Schmidt operators, finite rank perturbations of Volterra operators, infinite Leslie operators, discrete semi-separable operators, integral operators with semi-separable kernels, and period maps corresponding to delay differential equations. The classes of unbounded operators that are investigated appear in a natural way in the study of infinite dimensional dynamical systems such as mixed type functional differential equations, age-dependent population dynamics, and in the analysis of the Markov semigroup connected to the recently introduced zig-zag process. Complex Proofs of Real Theorems is an extended meditation on Hadamard's famous dictum, "The shortest and best way between two truths of the real domain often passes through the imaginary one." Directed at an audience acquainted with analysis at the first year graduate level, it aims at illustrating how complex variables can be used to provide quick and efficient proofs of a wide variety of important results in such areas of analysis as approximation theory,

operator theory, harmonic analysis, and complex dynamics. Topics discussed include weighted approximation on the line, Muntz's theorem, Toeplitz operators, Beurling's theorem on the invariant spaces of the shift operator, prediction theory, the Riesz convexity theorem, the Paley-Wiener theorem, the Titchmarsh convolution theorem, the Gleason-Kahane-Zelazko theorem, and the Fatou-Julia-Baker theorem. The discussion begins with the world's shortest proof of the fundamental theorem of algebra and concludes with Newman's almost effortless proof of the prime number theorem. Four brief appendices provide all necessary background in complex analysis beyond the standard first year graduate course. Lovers of analysis and beautiful proofs will read and reread this slim volume with pleasure and profit. The book provides a detailed exposition of the calculus of variations on fibre bundles and graded manifolds. It presents applications in such areas as non-relativistic mechanics, gauge theory, gravitation theory and topological field theory with emphasis on energy and energy-momentum conservation laws. Within this general context the first and second Noether theorems are treated in the very general setting of reducible degenerate graded Lagrangian theory. This is one of the first monographs to deal with the metric theory of spatial mappings and incorporates results in the theory of quasi-conformal, quasi-isometric and other mappings. The main subject is the study of the stability problem in Liouville's theorem on conformal mappings in space, which is representative of a number of problems on stability for transformation classes. To enable this investigation a wide range of mathematical tools has been developed which incorporate the calculus of variation, estimates for differential operators like Korn inequalities, properties of functions with bounded mean oscillation, etc. Results obtained by others researching similar topics are mentioned, and a survey is given of publications treating relevant questions or involving the technique proposed. This volume will be of great value to graduate students and researchers interested in geometric function theory. In 1918, Emmy Noether, in her paper *Invariante Variationsprobleme*, proved two theorems (and their converses) on variational problems that went on to

revolutionise theoretical physics. 100 years later, the mathematics of Noether's theorems continues to be generalised, and the physical applications of her results continue to diversify. This centenary volume brings together world-leading historians, philosophers, physicists, and mathematicians in order to clarify the historical context of this work, its foundational and philosophical consequences, and its myriad physical applications. Suitable for advanced undergraduate and graduate students and professional researchers, this is a go-to resource for those wishing to understand Noether's work on variational problems and the profound applications which it finds in contemporary physics. This volume presents a systematic and unified treatment of Leray-Schauder continuation theorems in nonlinear analysis. In particular, fixed point theory is established for many classes of maps, such as contractive, non-expansive, accretive, and compact maps, to name but a few. This book also presents coincidence and multiplicity results. Many applications. This English translation of my book "Priblizhenie Funktsii Mnogih Peremennyykh i Teoremy Vlozheniya" is identical in content with the Russian original, published by "Nauka" in 1969. However, I have corrected a number of errors. I am grateful to the publishing house Springer-Verlag for making my book available to mathematicians who do not know Russian. I am also especially grateful to the translator, Professor John M. Dancs, who has fulfilled his task with painstaking care. In doing so he has showed high qualifications both as a mathematician and as a translator of Russian, which is considered by many to be a very difficult language. The discussion in this book is restricted, for the most part, to functions everywhere defined in n -dimensional space. The study of these questions for functions given on bounded regions requires new methods. In connection with this I note that a new book, "Integral Representations of Functions and Imbedding Theorems", by O.V. Besov, V.P. Il'in, and myself, has just (May 1975) been published, by the publishing house "Nauka", in Moscow, U.S.S.R., May 1975. S.M. Nikol'skii Translator's Note I am very grateful to Professor Nikol'skii, whose knowledge of English, which is considered by many to be a very difficult language, is excellent, for much help in achieving a correct translation of

his book. And I join Professor Nikol'skir in thanking Springer-Verlag. The editing problem was considerable, and the typographical problem formidable. The Wiener-Wintner ergodic theorem is a strengthening of Birkhoff's pointwise ergodic theorem. Announced by N. Wiener and A. Wintner, this theorem has introduced the study of a general phenomenon in ergodic theory in which samplings are "good" for an uncountable number of systems. We study the rate of convergence in the uniform version of this theorem and what we call Wiener-Wintner dynamical systems and prove for these systems two pointwise results: the a.e. double recurrence theorem and the a.e. continuity of the fractional rotated ergodic Hilbert transform. Some extensions of the Wiener-Wintner ergodic theorem are also given. This volume contains a coherent point of view on various sharp pointwise inequalities for analytic functions in a disk in terms of the real part of the function on the boundary circle or in the disk itself. Inequalities of this type are frequently used in the theory of entire functions and in the analytic number theory. This book provides a comprehensive description of a new method of proving the central limit theorem, through the use of apparently unrelated results from information theory. It gives a basic introduction to the concepts of entropy and Fisher information, and collects together standard results concerning their behaviour. It brings together results from a number of research papers as well as unpublished material, showing how the techniques can give a unified view of limit theorems. The main purpose of writing this monograph is to give a picture of the progress made in recent years in understanding three of the deepest results of Functional Analysis—namely, the open-mapping and closed-graph theorems, and the so-called Krein-Mulian theorem. In order to facilitate the reading of this book, some of the important notions and well-known results about topological and vector

spaces have been collected in Chapter 1. The proofs of these results are omitted for the reason that they are easily available in any standard book on topology and vector spaces e.g. Bourbaki [2], Keiley [18], or Köthe [22]. The results of Chapter 2 are supposed to be well known for a study of topological vector spaces as well. Most of the definitions and notations of Chapter 2 are taken from Bourbaki's books [3] and [4] with some trimming and pruning here and there. Keeping the purpose of this book in mind, the presentation of the material is effected to give a quick resume of the results and the ideas very commonly used in this field, sacrificing the generality of some theorems for which one may consult other books, e.g. [3], [4], and [22]. From Chapter 3 onward, a detailed study of the open-mapping and closed-graph theorems as well as the Krein-Mulian theorem has been carried out. For the arrangement of the contents of Chapters 3 to 7, see the Historical Notes (Chapter 8). This is the second volume of the book on the proof of Fermat's Last Theorem by Wiles and Taylor (the first volume is published in the same series; see MMONO/243). Here the detail of the proof announced in the first volume is fully exposed. The book also includes basic materials and constructions in number theory and arithmetic geometry that are used in the proof. In the first volume the modularity lifting theorem on Galois representations has been reduced to properties of the deformation rings and the Hecke modules. The Hecke modules and the Selmer groups used to study deformation rings are constructed, and the required properties are established to complete the proof. The reader can learn basics on the integral models of modular curves and their reductions modulo p that lay the foundation of the construction of the Galois representations associated with modular forms. More background materials, including Galois cohomology, curves over integer rings, the Néron models of their Jacobians, etc., are also explained in the text and in the appendices.